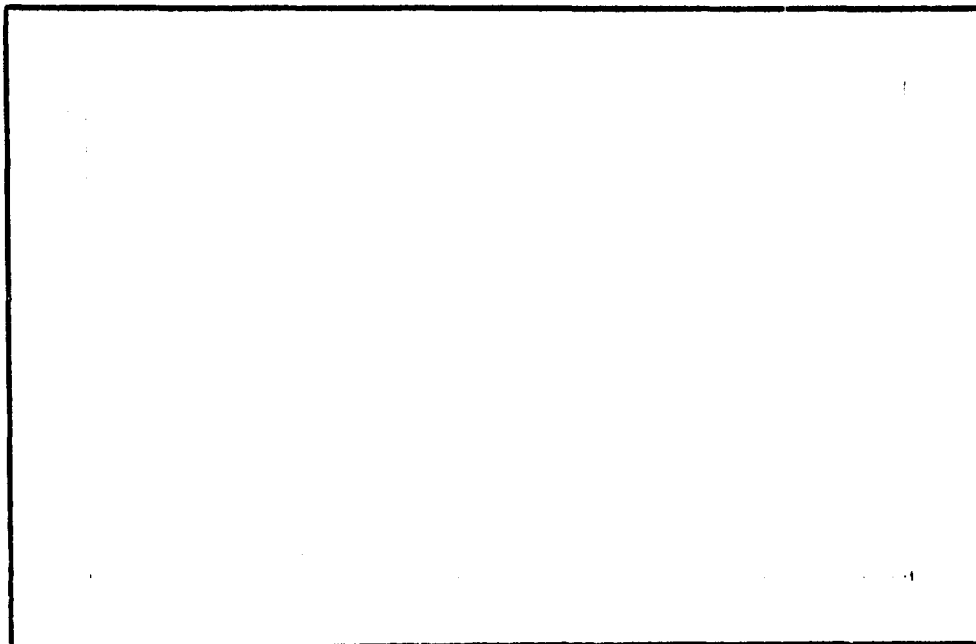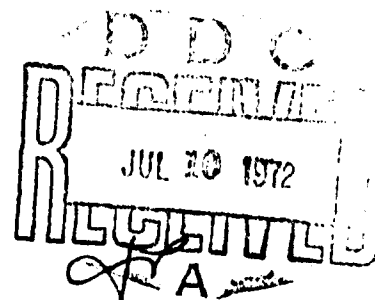# Carnegie-Mellon University
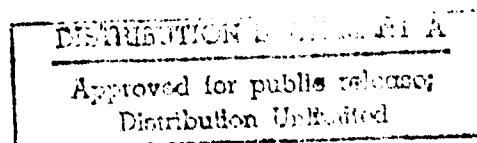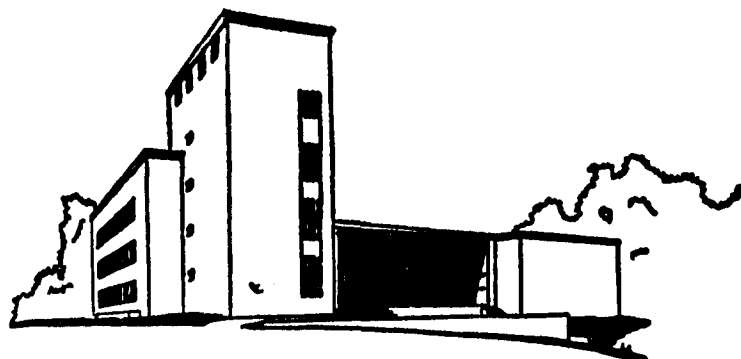
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AN ALGORITHM FOR MULTI-ATTRIBUTE

ASSIGNMENT MODELS

AND

SPECTRAL ANALYSES FOR DYNAMIC

ORGANIZATION DESIGN

by

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February, 1972

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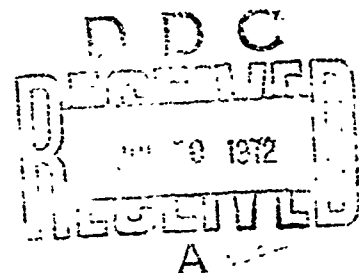
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13. ABSTRACT

A new algorithm for multi-attribute assignment models is presented. Built around the concept of a biased quadratic objective, it proceeds via a sequence of related assignments. Optional stopping is applied when an approximation to the optimum assignment is identified.

A second part of this paper deals with manpower planning and organization designs which interact dynamically. A spectral analysis approach is utilized to obtain estimates of career patters and aspirations in terms of observed and potential transition problems. Goal programming approaches are utilized in both parts of this paper. This is done not only to relate them to each other but also to relate them these deveopments to prior modeling efforts which the authors have undertaken in related areas of manpower planning. Implications for further research are sketched in areas such as the volunteer services and related kinds of organizations.

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1. Introduction:

The point of departure in this paper is our earlier "Static and Dynamic Assignment Models...." coauthored with A. Stedry [8.5]. That paper, ultimately, was directed to problems of dynamic organization design. En route to that objective, it dealt with extensions or ordinary assignment models which included extensions to multi-attribute characterizations of jobs and assignees. In motivation [8.5] was prompted by a recognition that organizations might be designed in response to the multi-attribute characteristics of its personnel raher than, as at present, attempting only to select and train personnel for static and pre-designed organization relations. The resulting organizations might then be further adjusted to changes, and even potential changes, in personnel characteristics. Organizations and even tasks to be performed could be further augmented to accomodate assignments that might be made not only in terms of (1) persons to jobs, but also to (2) decomposition and recombination of job possibilities, as well as (3) past and future contributions to learning from such job combinations along with (4) future experience and training possibilities.

The earlier paper [8.5] was largely directed to identifying and structuring these kinds of possibilities.[1] Its purpose was to open a way for further research and thereby contribute to progress in management science (and its relatives) as well as management practice (and its relatives). Beyond the indicated structurings, no attention was devoted to computational routines, data requirements, etc., of the sort that would be necessary for purposes of managerial implementation.

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[1]See [6] for further discussion of modeling techniques for identifying and structuring management problems.

This will be remedied here, if only in part, by initiating research in algorithmic developments for the assignment portions of the overall models. Note, however, that the usual assignment algorithms will not be adequate for the present case since the jobs and persons are both characterized in terms of sets of attributes. The usual scalar optimizations are then replaced by "goal-programming" varieties in which persons and job characteristics, which are multi-dimensional, are matched "as closely as possible."

As a start toward the wanted algorithmic developments we shall focus on the latter case. I.e., we shall develop our algorithmic suggestions around multi-attribute cases in which an optimization matches job requirements and personnel characteristics "as closely as possible." Extensions to the dynamics and other algorithmic suggestions, too, are not treated explicitly in this paper.

Many metrics may be selected for gauging the indicated "as close as possible."[1] Here, however, we shall focus on the "ℓ_1 metric" which in turn, corresponds to a use of absolute deviations. Via earlier research[2] the resulting nonlinear problem can be directly stated as a linear programming equivalent. The result is a "goal programming" model which provides access to other models of manpower planning, as in the OCMM series,[3] and also makes present computer codes available for assistance in developing the wanted algorithm.

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[1] See Appendix A in [4] for further discussion of these metric possibilities.

[2] See [5] and [4].

[3] See [8.0].

The above developments will be essayed in the first part of this paper in order, thereby, to sharpen aspects of the already developed "assignment model" extensions to the multi-attribute case. The second part will then be directed to extending some of the previously identified and structured topics in organization design--e.g., in relation to characteristics of available and potential personnel.

Many of the relevant personnel-dynamic organization design considerations are of a probabilistic nature, at best, and hence this, too, needs to be identified and structured in any comprehensive depiction of the problems and prospects for such designs. We do not propose to go into all aspects of these possibilities in the present paper. Instead we shall proceed discursively to identify salient aspects of the indicated possibilities while providing only so much formal structuring as will be needed to clarify intended meanings. To make matters more concrete we shall proceed in terms of a specific organization like, say, an all-volunteer armed service, or force, which one would like to organize in ways that will attract and develop the kinds of persons and relations needed for certain broadly defined "tasks."

The formalizations we shall essay will utilize some of the developments in recent research on mathematical programming approaches to "spectral analysis".[1] Naturally these developments will need to be modified and adapted for the purposes to be served. A natural next step in these developments will then be the creation of "control models" which deal with conformance between plans and relizations.[2] The form which

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[1] See [10] and [17].

[2] See [11] and [12]. See also [13].

we shall give to these further developments will be directed to relating planned and emergent assignments to longer term considerations of organization design and career management.

2. Algorithmic Developments for Multi-Attribute Assignments:

We initiate these algorithmic developments by reproducing the following definitions from the original multi-attribute extension of the assignment model from [8.5]. Therefore, let

$$x_{is} = \text{amount of individual s assigned to job i}$$

(1) $r_{ij} = \text{amount of } j\underline{^{th}} \text{ attribute required in job i}$

$$a_{sj} = \text{amount of } j\underline{^{th}} \text{ attribute possessed by individual s.}$$

The "assignment constraints": are

(2)

$$\sum_s x_{is} = 1$$

$$\sum_i x_{is} = 1$$

$$x_{is} = 0,1$$

The deviations from the goals may be formulated

(3) $\sum_s x_{is} a_{sj} - v_{ij}^+ + v_{ij}^- = r_{ij},$

where the "over" and "under" deviations are given by v_{ij}^+, $v_{ij}^- \geq 0$. A relevant objective is to minimize the under fulfillment of these goals via

(4) $\min \sum_i \sum_j v_{ij}^-$

The above formulation amounts, in essence, to minimizing the deviations from goals in a biased fashion. Of course, the deviations might additionally be weighted in a variety of ways, including, e.g., various uses of preemptive weights.[1]

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[1] See the discussion of Non Archimedean approaches in [8.5] as well as [7] and [14].

A practical difficulty with this formulation, as stated, is that it involves a mixed-integer programming problem. The ordinary assignment model, with linear objective function[1] consists of the system.

(5)
$$\min \sum_i \sum_s c_{is} \, x_{is}$$

subject to

$$\sum_i x_{is} = 1 \quad ,$$

$$\sum_s x_{is} = 1$$

$$x_{is} \geq 0.$$

We now observe that this system has the property that any extreme point solution--and a fortiori any optimum extreme point--automatically satisfies $x_{is} = 0,1$. Moreover this property holds even for preemptive or Non Archimedean c_{ij}.

For multi-attribute assignment models of any realistic size this "integer-solution (lattice point) property" can no longer be guaranteed. It therefore becomes desirable to consider new developments such as we shall here undertake in the form of biased-deviation formulations and sequential approximatories by means of sequences of ordinary linear programming assignment models. Another desideratum is to arrange matters so that at each sequential stage an approximate solution is at hand with all x_{is} at values of zero or unity.

Although the corresponding formulae can be easily developed for weighted (including preemptively weighted) as well as biased deviations, we shall, for simplicity, here develop only the case of uniform weights in explicit detail.

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[1] I.e., rather than a goal programming variety (with associated constraints.)

It is obviously desirable to formulate a goal deviation functional which is convex, and of a simple analytical form. This will result in a global optimum without local optima for the continuous problem in which the mixed integer problem is embedded. This, in turn, will also make it possible to achieve "close-to optimal" zero-one solutions by means of the sequence of approximating linear (possibly preemptive) assignment models.

3. Biased Quadratic Goal-Deviation Functional.

We start these developments with what we shall call a "biased, quadratic goal-deviation functional." We shall assume that the r_{ij} and a_{sj} are scaled so that $0 \leq r_{ij}, a_{sj} < 1$.

$$(6.1) \qquad z_{ij} \equiv \sum_s x_{is} a_{sj}.$$

Then a biased quadratic deviation from goal r_{ij} may be represented

$$\left(z_{ij} - r_{ij} \right)^2 - K_{ij} \left(z_{ij} - r_{ij} \right)$$

(6.2) where

$$K_{ij} \geq (1 - r_{ij}).$$

Graphically, this amounts to subtracting a linear function $y = K_{ij} \left(z_{ij} - r_{ij} \right)$ from the quadratic deviation function $(z_{ij} - r_{ij})^2$ so as to insure that values of z_{ij} exceeding r_{ij} are more desirable than values $z_{ij} \leq r_{ij}$. The choice of K_{ij} should be sufficiently large so that the linear function exceeds the quadratic function for the possible $z_{ij} > r_{ij}$.

Now, subject to the constraints,

$$(7.1) \qquad \begin{aligned} \sum_s x_{is} &= 1 \\ \sum_i x_{is} &= 1 \\ x_{is} &= 0,1 \end{aligned}$$

we wish to minimize

$$(7.2) \qquad Q(x,r) \equiv \sum_{i,j} \left[\left(\sum_s x_{is} a_{sj} - r_{ij} \right)^2 - K_{ij} \sum_s x_{is} a_{sj} \right].$$

Herein we have dropped the constant, $\sum_{ij} K_{ij} r_{ij}$, from the linear part since it does not affect the x_{ij} choices under minimization.

For the functional $Q(x,r)$, the gradient may be easily computed via

$$\frac{\partial Q}{\partial x_{is}} = \sum_{j} \left[2 \left(\sum_{t} x_{it} a_{tj} - r_{ij} \right) a_{sj} - K_{ij} a_{sj} \right]$$

(8)

$$= \sum_{t} x_{it} \left(\sum_{j} 2 a_{tj} a_{sj} \right) - \sum_{j} \left(2 r_{ij} + K_{ij} \right) a_{sj}$$

A sequence of iterations for linear assignment model solutions may now be started. The functional to be minimized at the $(m+1)^{\underline{st}}$ iteration is

$$(9) \qquad \sum_{i} \sum_{s} \frac{\partial Q}{\partial x_{is}}(n) \, x_{is}$$

where $x_{is}^{(n)}$ is the optimal extreme point solution[1] of the $n^{\underline{th}}$ iteration. Thus, the problem becomes, at this stage

$$\min \; \sum_{i} \sum_{s} \frac{\partial Q}{\partial x_{is}}(n) \, x_{is}$$

(10) with

$$\sum_{s} x_{is} = 1$$

$$\sum_{i} x_{is} = 1$$

$$x_{is} \geq 0$$

where, naturally, the solution gives $x_{is}^{(n)} = 0, 1$ since this is an ordinary assignment problem with a linear functional in the objective.

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[1] Hence $x_{is}^{(n)} = 0, 1$.

The procedure can be started with $x_{is}^{(o)} \equiv 0$ and the functional to be minimized as

(11) $\quad \sum_i \sum_s (-1) \sum_j \left(2 \, r_{ij} + K_{ij} \right) a_{sj} \, x_{is}$.

Even this first iteration, we observe, may give a good solution $x_{is}^{(1)} = 0, 1$.

From the above it should be clear how this method can be generalized to the case of arbitrarily (or even preemptively) weighted deviations. The result at each iteration will be an assignment model with linear functional. Hence it will have an optimal extreme point with coordinates $x_{is} = 0, 1$.

The above procedure, which is a gradient method operating (at each stage) on a bounded convex linear "flat" can be expected to converge to the face of the convex polytope which contains the optimum solution to the convex problem with the zero-one restrictions. At this juncture it may be expected to start oscillating between the extreme points (with 0,1 coordinates) which define the edges of the optimal face. These consist of the nearest zero-one points which are the solutions to the continuous convex (quadratic) assignment problem. By recording the value of the quadratic functional after each solution,[1] the progress can be compared and a stop rule devised. At the worst a practical stop rule can be provided in which computations halt when no improvement is obtained in a prescribed number of iterations. Experiments are in order, naturally, prior to the further development of this rule. We anticipate, however, that the zero[th] order iteration will, in fact, provide fairly good assignments in many cases.[2]

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[1] It might be noted that the value of the quadratic functional at the nth stage is given by

$$Q^{(n)} = \sum_{i,j} \left(z_{ij}^{(n)} - K_{ij} \right) \left(z_{ij}^{(n)} - r_{ij} \right)$$

[2] As was observed above.

4. Designs for Career Management and Career Aspirations: A Spectral Analysis Approach:

As was noted earlier, such algorithmic developments form only one part of the problems (and opportunities) we are concerned with identifying. Another part involves probabilistic extensions and new directions for studying some of the models that were previously structured (in [8.5]) for considering new kinds of dynamic organization designs. In particular, one may wish to consider the problem of deducing from the observed transition rates and populations--or equivalently the numbers transiting periodically (e.g., yearly)--what "careers" are being pursued and how many persons are pursuing them. For this purpose one may make an analogy between this problem and the spectral problem of resolving a univariate spectrum of energy (or light wave lengths or blood cell counts) into the proportions of elementary distributions (e.g., of Gaussian variety) of which the spectrum is composed. The overall spectrum for us is, however, multi-variate and consists of numbers of people (or transition rates) transiting from position to position in each year over a series of years, together with those occupying these various positions. The elementary distribution consists of "careers," or "truncated careers," which we take to be time sequences of positions occupied by a person (or a corresponding cohort) in an organization.

Within our time horizon we can consider as variables, the numbers of persons starting each career, or a truncated counterpart, at the beginning of each time period. In terms of these variables, knowing the fixed time sequence of jobs in a career, we can develop expressions for the numbers occupying various positions at the beginning of each

time period. The number of those occupying each position at the beginning of each time period can also be projected numerically from those initially aboard and the known transition rates. One can then use as the spectral analysis principle the objective of minimizing a weighted sum of the discrepancies between the "on-board" values projected by the variable numbers in the different careers and the numbers which emerge from the known transition rates and the initial job population. Such an analysis provides a new approach to career management based on an objective assessment through data which are independent of the subjective conceptions of careers being pursued by individuals in an ongoing organizational population. This analytical approach can thus provide aid to behavioral sciences research, and management, too, by systematically focusing on discrepancies between career aspirations and organizational realities, including the opportunities admitted by current organization designs.

One potential value which may be secured from this spectral analysis approach is that it can provide an evaluation of careers actually chosen and observed in terms of transitions actually effected. Differences between these objectively observed career patterns and those obtained via, e.g., personal interviews, can then be used to delinate areas for further research into organization design and personal interaction possibilities. Questions that may thus be raised include perception alterations as persons progress over time from the point at which they were introduced into the organization. This, in turn, may provide insight into organization alterations which can then suggest further possible changes in career pattern desires, and so on.

Of course, one must recognize that such expressions of career desires may not be well founded or even capable of complete articulation. This is likely to be an even more compelling consideration in the context of dynamically varying organization designs. In turn, this suggests a reorientation of objectives to allow for evolving contingencies of this variety. A stucture different from the Markoff transition varieties may be in order. For example, one might have changes in the demands for transfer from, and to, positions, depending on external or internal factors--including changes in aspirations which reflect changes in perceptions of alternative career patterns. To encompass such ideas the elements of transition matrices may need to be regarded as random variables. This, in turn, would excend the planning process to include planning for transition rates in which numbers of persons and resulting personnel mixes may be known only in probability. Recourse would then be had to models of chance-constrained (total or conditional) variety.

Suppose it is assumed that some control is possible in terms, e.g., of permitted transfers. It should then be possible to erect a conditional-adaptive variety of control model in which an organization can then try to match its task or job requirements by partially controlling (1) the emergent demands for transfers and (2) the new hires for entry into the system. Note that something akin to multi-attribute planning is also needed in these dimensions. The random character of the transition matrices means that these, too, form (partially controllable) elements of a total plan. Furthermore. the predictions will generally vary from the actuality which is subsequently encountered by both parties--viz., planners and participants. These discrepancies will evidently then need

to be considered in their different aspects for both planners and participants in only partially controllable form for any model which is devised for realistic applications to such situations.

5. Elements for a Spectral Analysis Model:

It is not possible to develop all of this at the present moment.
Indeed, a great deal of further research will be required to do this.
At this juncture it therefore seems best to try to sharpen some aspects
of the total problem. This will at least help to delineate what will
be required in a more comprehensive research effort. To that end we,
therefore, now focus on developing a spectral analysis type of model
which might be applied to the observable job transitions and the career
pattern provided by an existing organization. To state this differently,
we shall focus on the "objective" patterns provided by an existing
organization and ignore other aspects of personal aspiration and
organization redesign.

In one sense we can consider our spectral analysis model as involved
with modeling processes in which each process represents a career. Then
we can consider a career as a specified sequence of job positions
involving, say, a one-year duration for each position.

To represent this analytically let C_k designate career k via

$$(13) \qquad C_k = \left(J_k^1, \ J_k^2, \ \dots \ , \ J_k^n \right)$$

where J_k^s is the $s\underline{th}$ job occupied in the $k\underline{th}$ career type. We will be
considering a fixed period of years. Among the careers to be noted will
be some that might be called "truncated careers." Such truncations may
begin with a starting job which would only appear later in a non-
truncated career.[1] Thus,

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[1] It is also possible to extend the notion of career by considering
it as a sequence of jobs with a probability or proportional transition
from one job to another in a career sequence. We shall not develop this
extension here.

a truncation of C_k might be

$$(14) \qquad C_k^{'} = \left(J_k^2, \; J_k^3, \; \ldots, \; J_k^m \right).$$

We shall designate the jobs by numerals $i = 1, \ldots, m$. Thus the J_k^s may be considered as numbers in the range $i = 1, \ldots, m$.

Now suppose $x_k(r)$ represents the number of persons who choose career k in period r. Then, because of these career choices, the number in position i in year $s - 1$ is $f_i(s-1)$ where

$$(15) \qquad f_i \; (s-1) = \sum_{k:J_k^{s-1}=i} \; \sum_{r=1}^{s-1} \; x_k(r),$$

and the first summation designates the sum over careers k whose position in year s-1 is job i.

Let $f_{ij}(s)$ denote the number in position j in year s who were in position i in year s-1 by these choices. Then

$$(16) \qquad f_{ij} \; (s) = \sum_{k: \left(J_k^{s-1} = i; \; J_k^s = j \right)} \; \sum_{r=1}^{s=1} x_k(r) \quad .$$

Let

$$(17) \qquad M \equiv \left(M_{ij} \right)$$

denote the organizational transition matrix for the interval of years being considered. Then let $a_i(r)$ denote the number of persons in position i in year r prior to any hiring. Also let

$$(18) \qquad b_i \; (r) = a_i \; (r) + \text{new hires}.$$

The ratio $f_{ij}(s)/f_i(s-1)$ should approximate M_{ij} or, as a goal, we should like the discrepancy $\left| f_i(s-1) M_{ij} - f_{ij}(s) \right|$ to be as small as possible if the career patterns selected by the model are to have at least plausible normative validity. (The vertical strokes indicate an absolute value.)

Using the above considerations, we devise an objective which minimizes the indicated deviations via

$$(19) \qquad \min \sum_{i,j,s} \left| f_i(s-1) M_{ij} - f_{ij}(s) \right|.$$

This minimization is subject to the following constraints.[1]

$$(20) \qquad a_i(s) \le f_i(s) \le b_i(s)$$

where,[2] it may be recalled,

$$(21) \qquad f_i(s) = \sum_{k:J_k^s = i} \quad \sum_{r=1}^{s} x_k(r)$$

and $x_k(r) \ge 0$.

By virtue of previous research,[3] however, this spectral model (which is nonlinear) may be reformulated as a linear programming equivalent. Other normative principles of lesser detail can also be employed, of course, instead of the very precise correspondence implicit in the minimization indicated by (19).

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[1] It should be borne in mine that the $f_i(s)$ and $f_{ij}(s)$ are known linear functions of the variables $x_k(r)$.

[2] Observe that these constraints are in so-called "interval programming" form. See, e.g., [1] through [3].

[3] See [4] and [5].

6. Conclusion: Projections for Further Research:

In this paper we have regarded the elements M_{ij} of the Markoff matrix as fixed and known. Achieving the "least-deviation" x_k (r) values prescribed by the model then becomes a problem in actual personnel selection and development. Of course, managerial and technological, and other considerations, too, may also enter in an a priori fashion via stipulation of the upper and lower bounds which limit the x_k (r) choices in (20). In any event, additonal behavioral sciences research will undoubtedly be needed to insure that the persons selected will have the indicated career desires and aspirations.

Such reseach would constitute a rather natural extension of present practices in personnel practice insofar as personnel selection, career management, etc., are effected to conform "as closely as possible" to the likely career patterns (and related job satisfactions) admitted by a given organization.

The model suggested in this paper and its predecessors (see, e.g., [8] and [9], admit of other views which also invite attention. One such view would alter the status of the M_{ij} components in M and proceed to regard them as variables with values to be assigned in ways that conform (most closely) to the career aspirations and talents of the participants in an organization. In this view the x_k (r) can be regarded as given. The variables $M_{ij} \geq 0, \sum M_{ij} \leq 1$ would presumably then be submitted to organization and other constraints.[2] The latter comprehend task requirements as well as their admissible job decompositions. One could even extend these ideas and associate a series of

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[1] The same remark seems applicable to current practices in career management and analysis. Vide, e.g., the illustrative uses of a given Markoff matrix for career analysis and management described by Vroom and MacCrimmon in [16].

[2] Indeed, "goal programming" was originally invented for just such "constrained regression" uses. See, e.g., [4] and [5].

M_{ij} (t) in order to deal with various tasks that might be arranged for
an organization evolving over time.[1/] Personnel might then also be
related to each other for support or learning experiences that could
add to the opportunities afforded by a panorama of changing jobs and
career opportunities.

Discrepancies between empirically observed M_{ij} (t) and the
subjective career aspirations of organization members will naturally
invite attention. Analysis of such discrepancies may yield insights
and improved forecasts of quit rates or exits as well as new-entry
or voluntary reenlistments. Such analyses may also suggest organization
changes which will better tap the interests and energies of organization
participants.

All of the above point to further research which can be guided
and controlled by the kinds of spectral-analysis models we have portrayed
in this paper. This research can also be related to other possibilities,
too, which we have identified and structured in other papers in this
series. A case in point is the "generalized eigen value" approach
which we utilized in [9] to indicate how desired equilibrating
M_{ij} (t) may be arranged via recruitment, promotions, training, transfers,
etc. These roads to organization equilibria are, of course, also
pertinent for the present discussion. By advertising and via recruiting
for careers in a given organization, x_k (r) selections may be effected
to obtain the requisite numbers in each "career aspiration category"
to produce more desirable equilibria (or sequences of equilibria) in
the related M_{ij} (t). Success in the research that is thereby indicated
should then make it possible to deal simultaneously with manpower plan-
ning, career management and organization designs that are coordinated

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[1/]Of course the tasks that are forecasted or desired may be ordered
or weighted in a variety of ways to insure that desired goals are achieved
(as closely as possible) by specified dates, etc.

and directed toward producing better performance, better careers and greater job satisfactions in the organizations (military or civilian) which can then use these kinds of approaches.

All of the above, and more, may now begin to be effectual not only in response to the opportunities afforded by these new developments in modeling but also in response to the challenges of persons now demanding more rapid changes in organizatioi. that are better designed to attract and hold the efforts of volunteer participants.

This concludes the present paper. Data developments and numerical illustrations for some of these ideas are now being undertaken. These will be presented in some of the subsequent reports we are now readying for release in this OCMM series.[1]

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[1] Cf., the e.g., the reports noted under [8.0].

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